

# Minimizing 3-Phase THD in Inverters under Step Modulation

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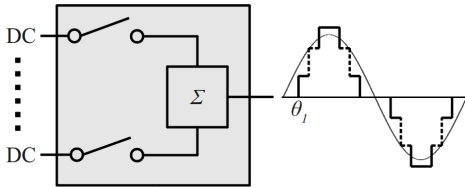
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## Abstract

Alternative energy sources such as solar panels often generate DC. However AC is needed for efficient transmission and for running appliances. Thus, one needs to produce AC from DC. One method is step modulation via switching. One main challenge is to determine the switching angles that minimize 3-phase total harmonic distortion (3THD) subject to a given modulation index. However, the expression for 3THD involves an infinite sum. Current techniques approximate the sum using only the first few terms. In this note we provide an equivalent finite expression for the sum and determine the optimal switching angles.

## I. Problem

One way to convert DC to AC is step modulation via switching [1,2] as shown in the following figure.



The 3-phase total harmonic distortion is defined as

$$D = \sqrt{\frac{\sum_{n=5,7,11,\dots} V_n^2}{V_1^2}}$$

where the sum is over all  $n$  that are odd but not a multiple of 3 and where

$$V_n = \frac{4}{\pi n} \sum_{k=1}^s \cos(n\theta_k)$$

The problem is a constrained optimization: minimize  $D$  subject to a given modulation index  $m = \frac{\pi}{4s} V_1$ .

## II. Solution

We provide an equivalent *finite* expression for  $D$ :

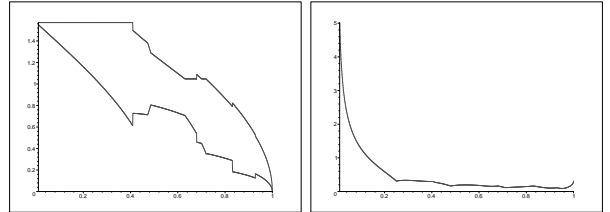
$$D = \sqrt{\left(\frac{4}{\pi V_1}\right)^2 \sum_{k_1=1}^s \sum_{k_2=1}^s C(\theta_{k_1}, \theta_{k_2}) - 1}$$

$$C(\alpha, \beta) = \frac{1}{2} (S(\alpha + \beta) + S(\alpha - \beta))$$

$$S(\phi) = T(\phi) - \frac{1}{3^2} T(3\phi)$$

$$T(\psi) = \begin{cases} -\frac{\pi}{4} (\psi - \frac{\pi}{2}) & \text{if } 0 \leq \psi \leq \pi \\ -T(\psi + \pi) & \text{if } \psi < 0 \\ -T(\psi - \pi) & \text{if } \psi > \pi \end{cases}$$

The figures below show the optimal switching angles (left) and the 3THD (right) for various modulation indices (2 DC sources for simplicity).



Note that the optimal angles are discontinuous. It is because 3THD is a piecewise differentiable function.

## References

1. J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, and Z. Du, *Elimination of harmonics in a multilevel converter using the theory of symmetric polynomials and resultants*. IEEE Trans. Control Systems Technology, vol. 13, pp. 216-223, 2005.
2. Y. Liu, H. Hong, and A. Q. Huang, *Real-time Calculation of Switching Angles Minimizing THD for Multilevel Inverters with Step Modulation*. IEEE Trans. Ind. Electron. vol. 56, no. 2, pp. 285-293, 2009.